Planar single-element gradient-index solar lenses for concentrator photovoltaics

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ABSTRACT

We present the design and simulation of first-ever planar single-element solar lenses (modified hemispherical gradient-index structures) for concentrator photovoltaic applications, with high collection efficiency and liberal optical tolerance at averaged cell irradiance levels exceeding 1000 suns. These compact lens designs satisfy the severe constraints of the refractive indices of viable polymeric materials and fabrication techniques, for visible and near-infrared radiation. The planar hemispherical gradient-index lens for a far-field (solar) source is created from a near-field unit magnification spherical gradient-index lens. Our new solutions incorporate a constant-index core (crucial for manufacturability). Simulations include a polychromatic and extended sun. A sample design for an f/1.40 solar lens is provided, where planar lenses comprise a concentrator module's protective glazing, with loss-less packing due to a square lens entry allowed by the modified truncated (non-full aperture) design, without incremental optical losses.

1. Introduction

Achieving ultra-high flux concentration with a single planar lens constitutes a pivotal challenge in solar concentrator photovoltaics (CPV), with the added value that such a lens can also serve as the glazing that encases the module. Efficient high-irradiance optical designs have been developed based on the simultaneous multiple surface method[1] and aplanatic optics[2], but necessitate two or more optical elements and often a non-planar entry aperture. Recent progress in the areas of (a) manufacturable polymeric gradient-index (GRIN) lenses[3,4] and (b) fundamentally new solutions for spherical GRIN lenses that can attain the basic limits for both image fidelity and concentration open heretofore unrecognized possibilities for solar concentration as well as for imaging visible light.

Here, we propose a novel class of manufacturable, planar hemispherical GRIN solar lenses. The solutions (GRIN profiles) presented are amenable to existing polymers and production techniques[3,4] (as opposed to the classic GRIN solutions derived by Maxwell[7] and Luneburg[8], which are not realizable for solar radiation due to the wide range of refractive indexes required).

Specifically, a *manufacturable* solar GRIN lens currently needs to satisfy[3,4]: (a) maximum and minimum refractive indices of n_{max} = 1.573 and n_{min} = 1.40, respectively, for transparent and extrudable polymeric materials, and (b) a spherical core, at least several millimeters in radius, of constant refractive index. In fact, no GRIN solutions that generate perfect imaging and incorporate an extended constant-index core had been discovered prior the publication of Ref 6.

The added degrees of freedom that proved crucial in identifying the GRIN solutions reported here are (1) an outer constant-index shell, and (b) a truncated (non-full) entrance aperture that does not reduce collection efficiency. The perfect-imaging maximum-concentration GRIN solutions published in Refs 5 and 6 were the first that could satisfy these constraints.

However, the GRIN solutions derived in Refs 5 and 6 are for *spherical* lenses. *Planar* GRIN lenses (that also do not compromise performance) would have the advantages of (1) being more compact and low-weight, (2) incurring lower reflection loss, (3) being simpler to manufacture[3,4], and (4) permitting square apertures that could basically eliminate packing losses within modules.

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2. Transforming spherical near-field GRIN lenses into planar hemispherical far-field lenses

We generate a planar lens for a *far*-field source by (a) designing a maximum-performance unit-magnification *near*-field GRIN lens, (b) noting that all incident paraxial rays are reconstituted as a collimated beam at the lens mid-section, and (c) eliminating one hemisphere while using the other hemisphere as a far-field planar concentrator. The GRIN solutions developed in Ref 6 were for the general near-field problem, and hence provide all the requisite mathematics for calculating these planar hemispherical GRIN lenses.

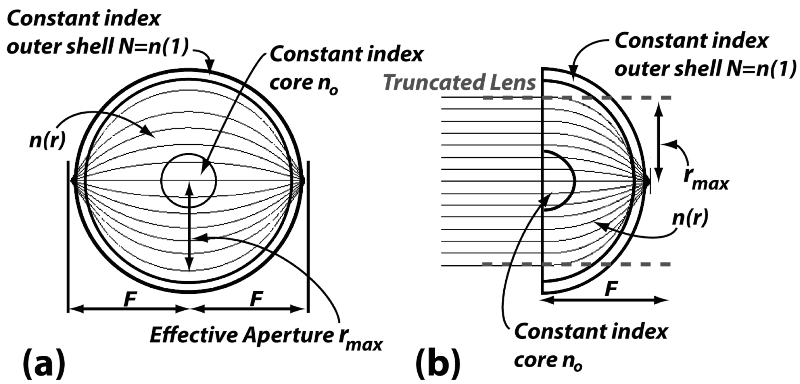

Fig 1. (a) Design of a hemispherical planar GRIN lens for a far-field (solar) source from a unit-magnification near-field design. (a) The unit-radius spherical GRIN lens has equal focal length F (measured from the center of the sphere) to the source and absorber. The extended spherical core has constant refractive index n_o. The outer shell has constant index N ($n(1)$). Between them is a continuous GRIN profile $n(r)$ (r denoting radial position in a unit-radius sphere). Several rays are traced from the source to the absorber. A truncated entry aperture at radial extent r_{max} can be introduced (elaborated in the text). (b) The lens is split at its mid-section, the left hemisphere is removed, and the remaining right hemisphere is a planar far-field concentrator that, as detailed in the text, is also aplanatic. A relatively low value of F was chosen for this illustration in order to emphasize the possibility of accommodating low f-number lenses, but this particular device does not satisfy the demanding fabrication and material constraints mentioned above. The planar hemispherical GRIN lens portrayed below *does* accommodate these constraints, but consequently possesses solutions only for larger values of F.

The specific illustrative example presented here was required to abide by (1) the manufacturing and material limitations described above for current technologies, and (2) providing flux concentration values of ~10^3 (typical of today's best CPV systems) at high collection efficiency and liberal optical tolerance. The GRIN solutions here (and in the related citations) are exact only at one radiation wavelength. No analytic generalizations to polychromatic radiation have yet been found. Raytrace simulation (LightTools, Synopsys, Inc.) was used to evaluate lens performance for both an extended (rather than point-source) sun of angular radius θ_{sun} = 0.005 rad (convolving concentrator and tracker optical errors with the inherent solar disc), and the complete AM1.5D solar spectrum.

3. Analytic solution and computational method

The analytic solutions derived in Ref 6 were adapted here to design a perfect-imaging unit-magnification (near-field) spherical GRIN lens with an extended constant-index core and constant-index exterior shell, as well as a truncated aperture:

$$n(\rho)=\begin{cases} N\exp\left[\begin{array}{l} 2\omega(\rho,F,A)+2\omega(\rho,N,A)-2\omega(\rho,1,A) \\ +\int_A^{A_s}\frac{f_1^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}}d\kappa+\int_{A_s}^{N}\frac{f_2^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}}d\kappa \end{array}\right], & 0\le\rho\le A \\ N\exp\left[\int_\rho^{A_s}\frac{f_1^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}}d\kappa+\int_{A_s}^{N}\frac{f_2^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}}d\kappa\right], & A\le\rho\le A_s \\ N\exp\left[\frac{1}{\pi}\int_\rho^{N}\frac{f_2^+(\kappa)}{\sqrt{\kappa^2-\rho^2}}d\kappa\right], & A_s\le\rho\le N \end{cases} \quad (1)$$

where $r(\rho)=\rho/n(\rho),\quad \omega(\rho,r,F)=\int_\rho^F\frac{\sin^{-1}(\kappa/r)}{\pi\sqrt{\kappa^2-\rho^2}}d\kappa,\quad A=r_{\max}n(r_{\max})$

where A_s denotes the product of n and r at the inner surface of the exterior constant-index shell. f_1^+ and f_2^+ are functions computed with the methods depicted in Ref 6, with $f_2^+=0$ over the interval $\{A_s,N\}$ for the constant-index exterior shell.

The *spherical* GRIN lens so designed is free of all orders of aberration (from a spherical source to a spherical absorber - referred to as exactly stigmatic[5,8]) for monochromatic light. Accounting for chromatic aberration is not generalizable, and is evaluated here by raytrace simulation. The fact that our absorber is planar rather than a spherical cap has a negligible influence, to wit, lowering concentration values of order 1000 suns by less than one sun.

The hemispherical planar GRIN lens depicted here is not only free of spherical aberration, but also incurs neither radial nor axial coma. It satisfies both Abbe's sine condition (that the sine of each ray's angle at the source is proportional to the sine of the same ray's angle at the absorber) and the Herschel condition (that the sines of the corresponding half-angles are proportional). This confluence is only possible for unit-magnification optics[9]. The proof of this assertion follows from an analysis with a point source and point absorber, which obviates the need to raytrace an extended source and absorber[9]. Conflating the perfect-imaging properties of the initial spherical GRIN lens with the observation that all rays from the near-field point source are exactly parallel at the flat entrance of the hemispherical lens that is formed from that spherical lens, we obtain a far-field lens that is aplanatic both radially and axially.

This planar hemispherical GRIN lens should approach the thermodynamic limit to flux concentration[1,2]

$$C_{max} = \{\sin(\theta_{out})/\sin(\theta_{in})\}^2 = A^2/\{F\sin(\theta_{sun})\}^2 \tag{2}$$

especially for effective θ_{sun} values as low as the 0.005 rad value adopted here. (θ_{out} denotes the maximum half-angle at the absorber.)

4. Illustrative example and simulated concentrator performance

By careful selection of the input parameters, the solutions of Eq (1) can possess a constant refractive index in both an extended core *and* exterior shell. (As detailed in Ref 6, the core index is not rigorously constant, but rather varies by no more than about ±0.001, which results in negligible aberration.) One starts with a guess n_o^* for the core's constant index, and follows the computational method to arrive at the core index n_o consistent with the governing integral equations and boundary conditions. The illustrative example here (see Figs 2-4) has $N = 1.415$, $A = 0.97$, $A_s = 1.365$ and $n_o^* = 1.573$ - values geared toward CPV applications at $C = 1100$, with cell diameters of ~1 mm (not uncommon in current CPV systems), for which the lens diameter would be ~33 mm. Furthermore, designing for $r_{max} < 1/\sqrt{2}$ permits the lens entrance to be square, thereby avoiding packing losses within modules (see Fig 4).

Lens performance was simulated by raytracing, including the dispersion properties of current viable polymers[3,4] (chromatic aberration for the broad-spectrum solar input is included in Figs 3-4). Absorption in the lens and reflections from its exterior surfaces are not included because they are material-specific, reasonably small, and straightforward to estimate.

As for tolerance to angular deviations from on-axis orientation (commonly quantified as the angular deviation at which no more than 10% of collectible radiation is rejected relative to proper alignment), the fundamental bound[10] (corresponding to 0.010 rad here, for $\theta_{sun} = 0.005$ rad, $\theta_{out} = 0.5236$ rad and $C = 1100$) is reached with monochromatic radiation, but a tolerance of only 0.005 rad is attained for the full polychromatic solar source.

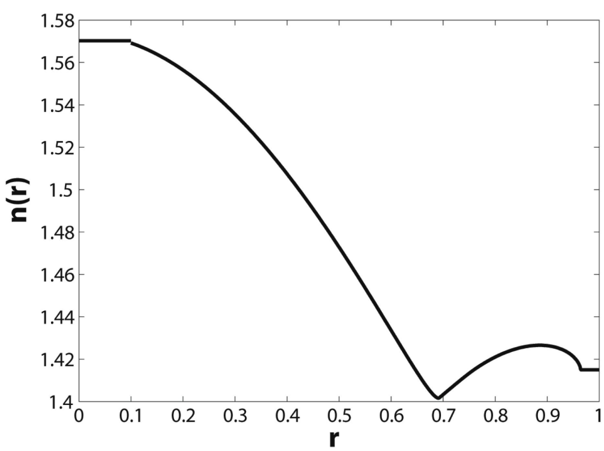

Fig 2. *n(r)* for a hemispherical planar GRIN lens with $r_{max} = 0.693$ and $F = 1.94$ (a *f*/1.40 lens with $\theta_{out} = 0.5236$ rad). There is a core region of constant refractive index $n_o = 1.573$ extending up to $r = 0.1$, and a continuous GRIN profile between that core and a constant-index exterior shell. While the *n(r)* function possesses extrema, the natural variable in the governing integral equations, $\rho = rn(r)$, is a monotonically increasing function of r, as required by the formalism[6]. Although C_{max} (Eq (2)) is 10000 here, the lens is intended for CPV at $C = 1100$.

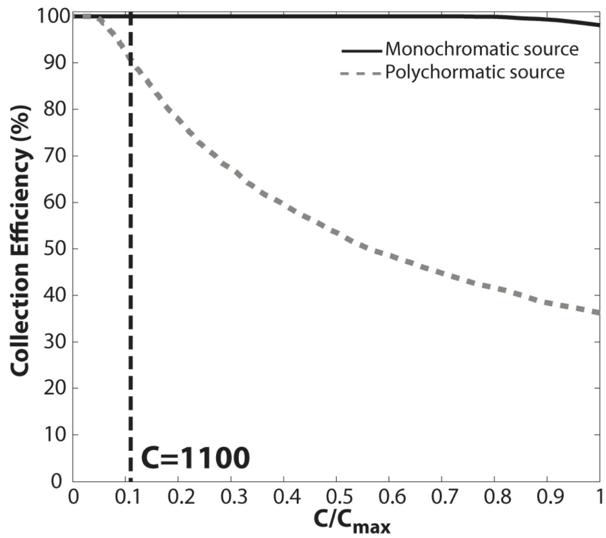

Fig 3. Geometric collection efficiency plotted against concentration C relative to the thermodynamic limit C_{max} of Eq (2). These results were generated by raytrace simulation with an extended far-field light source of angular radius 0.005 rad, for both monochromatic (red) and broadband (AM1.5D) radiation. The vertical dashed line indicates the value $C = 1100$ common to current CPV.

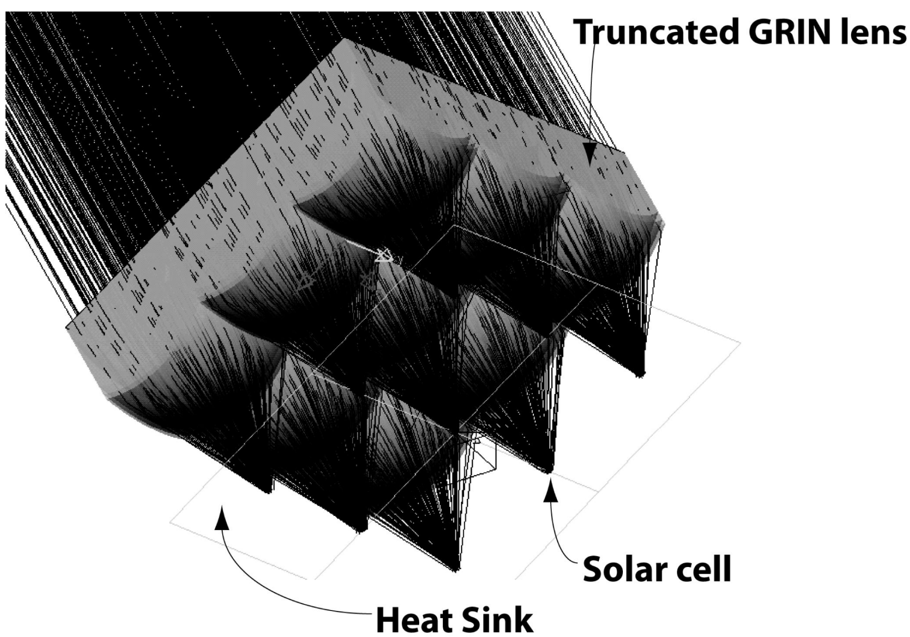

Fig 4. Raytrace results (with a broadband AM1.5D spectrum and extended solar source of angular radius 0.005 rad) for a group of truncated planar hemispherical GRIN lenses that also constitute a CPV module's entrance glazing. Packing losses are basically eliminated as a consequence of the square entrance aperture permitted by the truncated design method described above.

5. Summary

A judiciously modified near-field unit-magnification fully spherical GRIN lens can spawn a far-field planar hemispherical GRIN lens that closely approaches the thermodynamic limit to concentration in a single optical element. The solutions (GRIN profiles) noted here satisfy the severe realistic constraints of using only existing transparent extrudable polymers and associated production techniques, the latter including the requirement of an extended constant-index core to the lens.

The sample lens illustrated here provides near-perfect imaging and near-maximum concentration for monochromatic radiation, (when tracking a far-field source) and, as confirmed by raytracing, maintains high performance even for the actual broadband and angularly extended solar source. The realistic constraints on refractive index values result in focal lengths F that are closer to 2 than to 1. At such time as materials and manufacturing procedures become available that can reduce n_{min} and increase n_{max}, GRIN solutions with noticeably lower f-numbers emerge (e.g., Fig 1) that are (1) more compact and (2) can attain higher concentration at a given acceptance angle or, alternatively, can improve optical tolerance at fixed concentration (Eq (2)). Finally, it might be noted that interchanging the source and target allows the planar hemispherical GRIN lenses derived here to be deployed for collimating light-emitting diodes.

Acknowledgement

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