Realizable planar gradient-index solar lenses

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Received December 19, 2011; revised January 31, 2012; accepted January 31, 2012; posted January 31, 2012 (Doc. ID 160233); published March 27, 2012

The design of single element planar hemispherical gradient-index solar lenses that can accommodate the constraints of realistic materials and fabrication techniques are presented, and simulated with an extended and polychromatic solar source for concentrator photovoltaics at flux concentration values exceeding 1000 suns. The planar hemispherical far-field lens is created from a near-field unit magnification spherical gradient-index design, and illustrated with an $f/1.40$ square solar lens that allows lossless packing within a concentrator module.

OCIS codes: 110.2760, 350.6050, 220.1770.

Is ultrahigh solar concentration attainable with just a single planar optical element? The answer is especially relevant for concentrator photovoltaics (CPV) where a planar lens can serve double duty as the module's protective glazing. Furthermore, a lens inherently permits an upward facing absorber, markedly facilitating unobstructive passive heat sinks. Simultaneous multiple surface [1] and aplanatic [2] optics can indeed provide high flux densities efficiently, but require multiple optical elements and often contoured apertures. The confluence of recent advances in (1) manufacturable gradient-index (GRIN) polymer lenses [3,4] and (2) novel design techniques for spherical GRIN lenses that perform at the fundamental limits for flux concentration and imaging [5,6] create new capabilities for imaging and concentrating visible and solar radiation.

This Letter describes the design and simulation of realizable, planar hemispherical GRIN solar lenses. By realizable, we refer to solutions that accommodate existing polymer materials and manufacturing procedures [3,4]—in contrast to the pioneering GRIN solutions of Maxwell [7] and Luneburg [8] which, for visible and solar frequencies, are infeasible because of the broad range of refractive indices required.

To wit, the constraints for current realistic GRIN solar lenses are [3,4] (1) minimum and maximum refractive index values of $n_{\min} = 1.40$ and $n_{\max} = 1.573$, respectively, for transparent polymers that can be extruded into GRIN structures, and (2) an extensive (e.g., several mm radius) constant-index core is required for the fabrication process. (No perfect imaging GRIN solutions had been found for the latter prior to [6].) Two extra degrees of design freedom that proved distinctly valuable in allowing the solutions identified here are (a) an outer homogeneous shell and (b) a nonfull (truncated) entry at no loss in light collection. Prior to the solutions presented in [5,6], no GRIN profiles capable of producing nominally perfect imaging and efficient maximum concentration that could accommodate these constraints had been identified.

The solutions in [5,6] relate to spherical GRIN lenses. The advantages of planar GRIN lenses capable of comparable performance would be (a) lower mass and superior compactness, (b) lower Fresnel reflective losses (the lenses could serve as the module's protective glazing), and (c) easier manufacture [3,4], all while allowing square entries that obviate lens packing losses.

The strategy (Fig. 1) is to create a planar lens for a far-field source by (1) designing a nominally perfect imaging near-field lens of unit magnification, (2) recognizing that all on-axis rays must traverse the lens midsection as a collimated beam, and (3) removing half the lens and deploying the remaining hemisphere as a planar far-field concentrator. Because the solutions in [6] were derived for the general near-field problem, all the mathematical tools are available for adaptation to the planar lens notion.

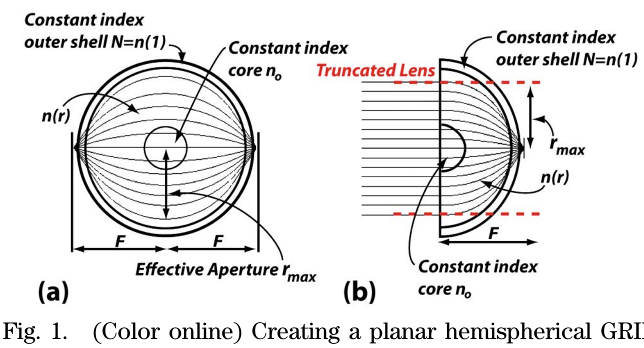

Fig. 1. (Color online) Creating a planar hemispherical GRIN lens for a far-field source from a near-field unit magnification spherical GRIN lens. (a) The spherical lens (of unit radius) has equal object (source) and image (absorber) focal length F (measured from the sphere's center), a core of constant index n_o, a homogeneous outer shell of index $N = n(1)$, and a GRIN continuum $n(r)$ between them (r denoting radial position), including a trace of several rays. A nonfull aperture is allowed such that $r_{\max} < 1$ (*vide infra*). (b) When the lens is split and only the hemisphere retained, it becomes a far-field planar aplanatic concentrator. This illustration was purposely chosen to highlight the potential for short focal lengths, but does not fulfill the severe material and manufacturing constraints noted above. Because the illustrative example below does impose these limitations, it possesses physically admissible solutions only for noticeably larger F.

For economy of presentation, the illustrative example below is forced to comply with (a) the material and fabrication constraints of realizable GRIN lenses noted above, and (b) delivering efficient flux concentration commensurate with current CPV (i.e., $\sim 10^3$) at liberal optical error. The solutions are rigorous only for monochromatic radiation, with no known analytic generalizations for broadband light. Lens performance is evaluated here by ray tracing (LightTools, Synopsys, Inc.), and incorporates both the full solar spectrum (AM1.5D) and an extended (rather than point) source, taken here as an effective angular radius $\theta_{\rm sun} = 5$ mrad comprising a convolution of the intrinsic solar disc and system optical errors, with conventional CPV dual-axis solar tracking.

Adapting results derived in [6], the analytic solution to the governing integral equations for the index profile of a spherical GRIN lens that produces perfect near-field imaging—specifically, when a core of constant index is mandated, with an outer homogeneous shell of index $N = n(1)$, an effective (nonfull) aperture with $A = r_{\max} n(r_{\max})$, and unit magnification is

$$n(\rho) = \begin{cases} N \exp\left[2\omega(\rho,F,A) + 2\omega(\rho,N,A) - 2\omega(\rho,1,A) + \int_A^{A_s} \frac{f_1^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa + \int_{A_s}^N \frac{f_2^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa\right], & 0 \le \rho \le A \\ N \exp\left[\int_\rho^{A_s} \frac{f_1^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa + \int_{A_s}^N \frac{f_2^+(\kappa)}{\pi\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa\right], & A \le \rho \le A_s \\ N \exp\left[\frac{1}{\pi}\int_\rho^N \frac{f_2^+(\kappa)}{\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa\right], & A_s \le \rho \le N \end{cases},$$

where

$$r(\rho) = \rho/n(\rho), \omega(\rho,r,F) = \int_\rho^F \frac{\sin^{-1}(\kappa/r)}{\pi\sqrt{\kappa^2-\rho^2}} \mathrm{d}\kappa \qquad (1)$$

out the need to trace rays from the actual extended source [9]. The combination of the perfect imaging of the parent spherical GRIN lens with the symmetry of unit magnification ensures that all point source rays are precisely parallel to the optic axis at the planar entry of the hemispherical lens created from it, thereby forming a far-field lens that is aplanatic both radially and axially.

Such a lens might be expected [2] to closely approach the appropriate thermodynamic limit to concentration [1]

$$C_{\max} = \{\sin(\theta_{\rm out})/\sin(\theta_{\rm sun})\}^2 = A^2/\{F\sin(\theta_{\rm sun})\}^2 \qquad (2)$$

for acceptance angles $\theta_{\rm sun}$ as small as those considered here ($\theta_{\rm out}$ is the maximum exit half angle at the absorber).

For judiciously selected input parameters, the solution to Eq. (1) can yield a constant-index core (as explained in [6], not strictly constant, but constant to within no worse than about $\pm 10^{-3}$, which introduces negligible aberration), and a GRIN continuum between the core and the homogeneous outer shell. The computation requires

and A_s is the product of r and n at the interior of the outer homogeneous shell. The functions f_1^+ and f_2^+ can be computed using the procedures detailed in [6], with f_2^+ being zero on the interval $\{A_s, N\}$ in the case of a homogeneous outer shell.

This *spherical* GRIN lens is exactly stigmatic for monochromatic radiation, i.e., devoid of all orders of aberration from a spherical-cap object to a spherical-cap image [5,8]. Chromatic aberration is case specific and must be quantified by ray tracing (*vide infra*). Distortion from a spherical cap to a planar absorber is insignificant (i.e., lowering flux concentration by less than one part in 10^3) at concentration values of order 10^3.

The planar hemispherical GRIN lens portrayed here is free of both radial and axial comma (in addition to incurring no spherical aberration). Namely, it respectively satisfies both the Abbe sine condition (proportionality of the sine of a given ray's angle at the source and target) and the Herschel sine condition (proportionality of the sine of the corresponding half angles)—a possibility unique to unit magnification optics [9]. Both conditions can be established based on a point source analysis, i.e., with-

an initial guess n_o^* of the constant core index and generates the actual core index n_o. The numerical values used in the example are $N = 1.415$, $A = 0.97$, $A_s = 1.365$ and $n_o^* = 1.56$ (yielding a solution with $n_o = 1.573$) (Fig. 2)—intended for CPV at a concentration of 1100, such that a cell linear dimension of 1 mm (now common in CPV) would correspond to a lens diameter of ~33 mm. Solutions with $r_{\max} < 1/\sqrt{2}$ allow a square lens entry and obviate lens packing losses (Fig. 4 below).

The polychromatic performance of the lens (Fig. 3) is assessed with ray tracing, based on the relevant polymer dispersion characteristics [3,4] (also see Fig. 4). Lens absorption and Fresnel reflections are not incorporated since they are typically material specific, small, and straightforward to evaluate. Chromatic aberration results in only near-negligible losses at the target concentration of 1100 (the fact that the center of the lens focal spot contains a concentration of 10,000 with a nonnegligible fraction of collectible radiation augurs for its possible value in pragmatic solar-driven materials testing).

Regarding tolerance to off-axis orientation (conventionally expressed as the angular deviation up to which

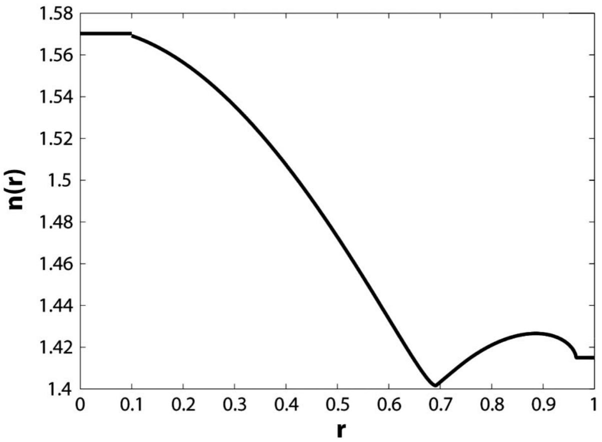

Fig. 2. Refractive index distribution for a planar hemispherical GRIN lens with $F = 1.94$ and $r_{\max} = 0.693$ ($f/1.40$, $\sin(\theta_{\text{out}}) = 0.5$). The constant-index core, with $n_o = 1.573$, extends up to $r = 0.1$. A single GRIN continuum connects the core and outer shell. Although the GRIN region exhibits extrema for $n(r)$, the natural variable in the governing equation, $\rho = rn(r)$, is actually a monotonically increasing function of r, as required by the formalism [6]. The corresponding $C_{\max}$ is 10,000 [Eq. (2)], and the lens is intended for CPV at $C = 1100$.

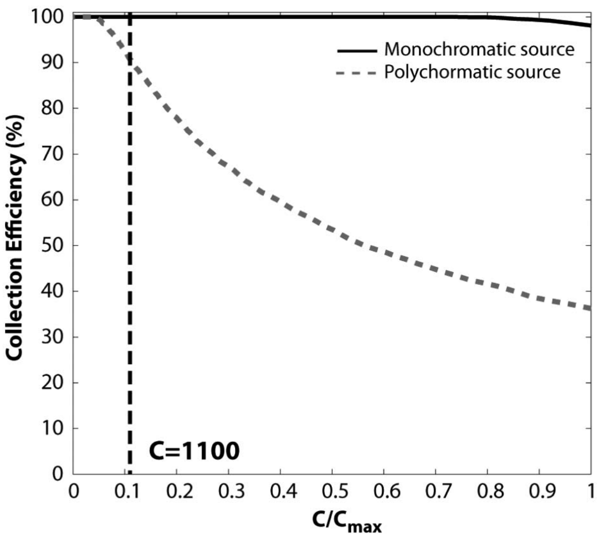

Fig. 3. Geometric collection efficiency as a function of flux concentration C [relative to the thermodynamic limit $C_{\max}$, Eq. (2)] evaluated by ray tracing with an extended source of 5 mrad angular radius, for both monochromatic (red) and polychromatic (AM1.5D spectrum) light. $C = 1100$, common to current CPV, is indicated by the vertical dashed line.

90% of on-axis collection is maintained), ray tracing reveals that the fundamental bound [10] of 10 mrad (at $\theta_{\text{sun}} = 5$ mrad, $C = 1100$, and $\theta_{\text{out}} = 30°$) is realized with monochromatic light, but only half that value is achieved with the full solar spectrum. If needed, concentration and/or off-axis performance can be enhanced by optically bonding a terminal concentrator/flux homogenizer to the solar cell [1].

Achieving a realizable planar far-field hemispherical GRIN lens by modifying an ideal near-field fully spherical GRIN lens necessitates GRIN solutions that can satisfy the stringent constraints on refractive index values im-

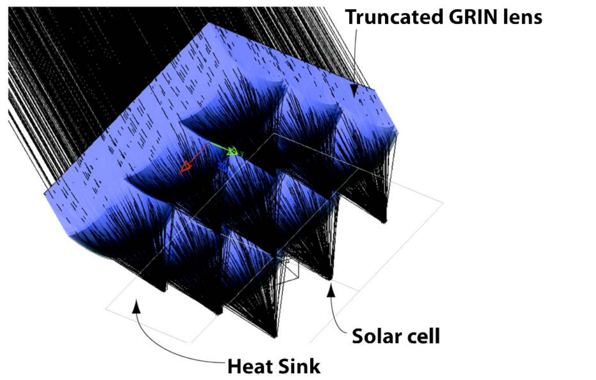

Fig. 4. (Color online) Ray trace (with a polychromatic and extended sun) illustrating the performance of a set of planar lenses that comprise part of the module's protective glazing, with lossless packing due to a square entry allowed by the truncated design.

posed by the conflation of existing transparent polymer materials and the extrusion techniques for molding them into GRIN layers, including the requirement of an extensive constant-index core. The illustrative example presented is a rigorous solution based on monochromatic light, and turns out to be robust for the full solar spectrum and an extended (as opposed to a point) source. A focal length closer to 2 than to 1 is required to fulfill the acute constraints. Should new materials and production methods become available—that can lower $n_{\min}$ and raise $n_{\max}$—then far lower $f\#$ lenses have admissible solutions (e.g., Fig. 1), with the concurrent advantages of (a) being more compact and (b) higher concentration at fixed acceptance angle or allowing greater optical errors (effective θ_{sun}) for the same concentration [Eq. (2)]. Although not elaborated here, these hemispherical GRIN solutions can be used in reverse to provide compact high-performance collimation for light emitting diodes.

This research was funded by the Defense Advanced Research Programs Agency, under the Manufacturable Gradient Index (M-GRIN) program, contract no. HR0011-10-C-0110.

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